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HW 4.5 Graphing Cosine and Cosecant and Cotangent Function with Transformations

1. Indicate if the following formulas are true for all values of θ:

i) $\cos\theta = -\cos\theta$ ✗

ii) $\sin\theta = -\sin\theta$ ✗ ✓

iii) $\cos\theta = \cos(-\theta)$ ✓

iv) $\tan\theta = -\tan(-\theta)$ ✓

vi) $\sec\theta = \dfrac{1}{\cos(-\theta)}$ ✓

vii) $\sin\theta = \dfrac{\sin 2\theta}{2\cos\theta}$ ✓ $\sin\theta = \frac{2\sin\theta\cos\theta}{2\cos\theta}$

viii) $\sin\theta = \sin(\pi - \theta)$ ✓

ix) $\tan(\theta_1 + \theta_2) = \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}$ ✓

2. Find a general formula for the asymptotes, find domain, range, and period.

asymptote increments by half of period.

a) $y = 2\csc\left(\theta + \frac{\pi}{3}\right) - 2$

$y = 2\csc\left(\theta + \frac{\pi}{3}\right) - 2$

$\sin\left(\theta + \frac{\pi}{3}\right) = 0$

$\theta = -\frac{\pi}{3}$

Domain: $x \in \mathbb{R}; x \neq \frac{\pi}{3} + n\pi$

Range: $y \leq -4$ or $y \geq 0$

Period $= 2\pi$ | Asymptote: $x \neq -\frac{\pi}{3} + n\pi; n \in \mathbb{Z}$

b) $y = -2\sec\left(\theta + \frac{2\pi}{3}\right) - 1$

$y = -2\cos\left(\theta + \frac{2\pi}{3}\right) - 1$

$\cos\left(\theta + \frac{2\pi}{3}\right) = 0$

$\theta = -\frac{\pi}{6}$

D: $x \in \mathbb{R}; x \neq -\frac{\pi}{6} + n\pi$

R: $1 \leq y$ or $y \leq -3$

Period: 2π, Asymptote: $x = -\frac{\pi}{6} + n\pi; n \in \mathbb{Z}$

c) $y = -3\tan\pi(-2\theta + 3) + 4$

Asymptotes usually at $\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$y = 3\tan\pi(2\theta - 3) + 4$

Period: $\frac{\pi}{2\pi} = 0.5^R$

Asymptotes $4.25 + 0.5n; n \in \mathbb{Z}$

D: $x \in \mathbb{R}; x \neq 4.25 + 0.5n; n \in \mathbb{Z}$

R: $y \in \mathbb{R}$

d) $y = -\cot\left(2\theta - \frac{\pi}{5}\right) - 4$

$y = -\cot\left[2\left(\theta - \frac{\pi}{10}\right)\right] - 4$

Period: $\frac{\pi}{2}$

Asymptotes: $\frac{\pi}{10} + \frac{\pi}{2}n; n \in \mathbb{Z}$

D: $x \in \mathbb{R}; x \neq \frac{\pi}{10} + \frac{\pi}{2}n; n \in \mathbb{Z}$

R: $y \in \mathbb{R}$

e) $y = 3\csc\frac{\pi}{4}\left(2\theta + \frac{2}{3}\right) - 2$

$y = 3\csc\left[\frac{\pi}{2}\left(\theta + \frac{1}{3}\right)\right] - 2$

Period: $\frac{2\pi}{\frac{\pi}{2}} = 4^R$

Asymptote: $-\frac{1}{3} + 2n^R; n \in \mathbb{Z}$

D: $x \in \mathbb{R}; x \neq -\frac{1}{3} + 2n^R; n \in \mathbb{Z}$

R: $y \geq 1$ or $y \leq 5$

f) $y = -3\sec\frac{3\pi}{8}(3\theta + 2) + 1$

$y = -3\sec\left[\frac{3\pi}{8}\left(\theta + \frac{2}{3}\right)\right] + 1$

Period: $\frac{2\pi}{\frac{3\pi}{8}} = \frac{16}{9}$

Asymptote: $\frac{10}{9} + \frac{8}{9}n; n \in \mathbb{Z}$

D: $x \in \mathbb{R}; x \neq \frac{10}{9}^R + \frac{8n}{9}; n \in \mathbb{Z}$

R: $4 \leq y$ or $y \leq -2$

g) $y = 3\tan\left(5x - \frac{\pi}{3}\right) + 4$

$y = 3\tan\left[5\left(x - \frac{\pi}{15}\right)\right] + 4$

Period: $\frac{\pi}{5}$

Asymptote: where the function starts + $\frac{\text{period}}{4}$

$= \frac{\pi}{15} + \frac{\pi}{10} + \frac{\pi n}{5}$; $n \in \mathbb{Z}$

$= \frac{\pi}{6} + \frac{\pi n}{5}$; $n \in \mathbb{Z}$

D: $x \in \mathbb{R}$; $x \neq \frac{\pi}{6} + \frac{n\pi}{5}$; $n \in \mathbb{Z}$

R: $y \in \mathbb{R}$.

h) $y = 5\cot\pi\left(3x - \frac{7}{6}\right) - 5$

$y = 5\cot\left[3\pi\left(x - \frac{7}{18}\right)\right] - 5$

Period: $\frac{\pi}{3\pi} = \frac{1}{3}^R$

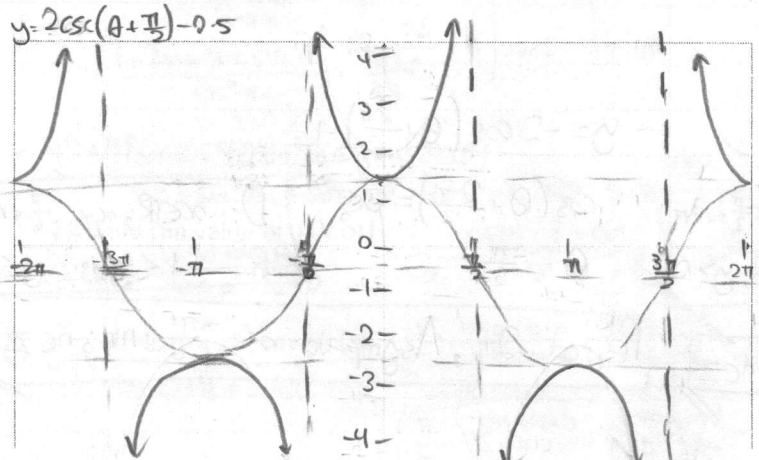
Asymptote: $\frac{7}{18}^R + \frac{1}{3}n^R$; $n \in \mathbb{R}$

D: $x \in \mathbb{R}$; $x \neq \frac{1}{3}n^R$

R: $y \in \mathbb{R}$

3. Graph the corresponding Sine/Cosine function, and then graph the equation: $y = 2\csc\left(\theta + \frac{\pi}{2}\right) - 0.5$

$y = 2\sin\left(\theta + \frac{\pi}{2}\right) - 0.5$



Indicate domain and range:

D: $x \in \mathbb{R}$; $x \neq \frac{\pi}{2} + n\pi$; $n \in \mathbb{Z}$

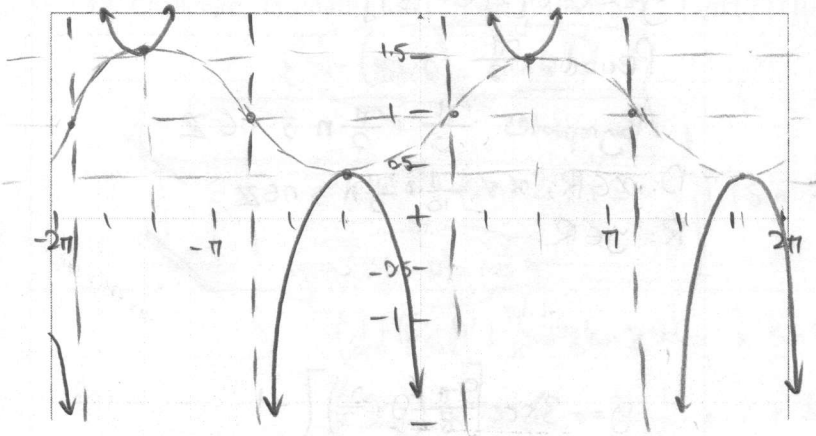
R: $y \geq 1.5$ or $y \leq -2.5$

Find a general formula for all Vert. Asymptote

$x = \frac{\pi}{2} + n\pi$; $n \in \mathbb{Z}$

4. Graph the corresponding Sine/Cosine function, and then graph the equation: $y = 0.5\sec\left(\theta - \frac{2\pi}{3}\right) + 1$

$y = 0.5\cos\left(\theta - \frac{2\pi}{3}\right) + 1$



Indicate the domain and range:

D: $x \in \mathbb{R}$; $x \neq \frac{7\pi}{6} + \pi n$; $n \in \mathbb{Z}$

R: $y \in \mathbb{R}$

Find a general formula for all Vert. Asymptote

$x \neq \frac{7\pi}{6} + \pi n$; $n \in \mathbb{Z}$

5. Find the period, amplitude, and phase shift of $y = -2\sin\left(\frac{x}{4} - \frac{\pi}{3}\right)$

$y = -2\sin\left[\frac{1}{4}\left(x - \frac{4\pi}{3}\right)\right]$

Period: $\frac{2\pi}{\frac{1}{4}} = 8\pi$

Amplitude: 2

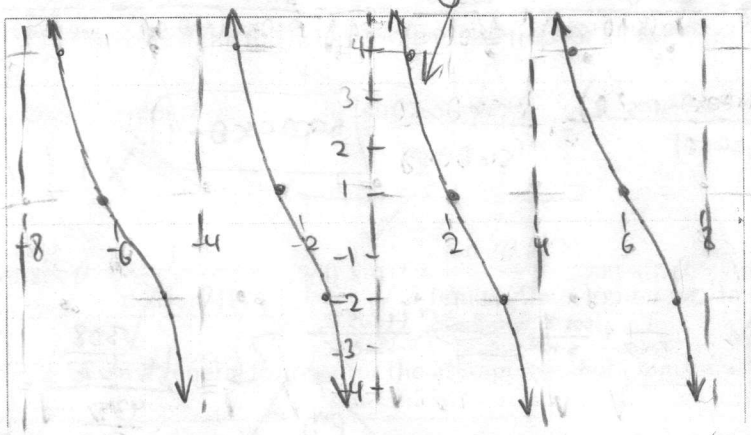
Phase shift: $\frac{4\pi}{3}$

6. Graph the function on the graph provided: $y = 3\tan\frac{\pi}{4}(-\theta+2)+1$

Negative tan!!!

$y = -3\tan\frac{\pi}{4}(\theta-2)+1$

Period: $\frac{\pi}{\frac{\pi}{4}} = 4^R$



Indicate the domain and range:

D: $x \in \mathbb{R}$, $x \neq 4n$, $n \in \mathbb{Z}$

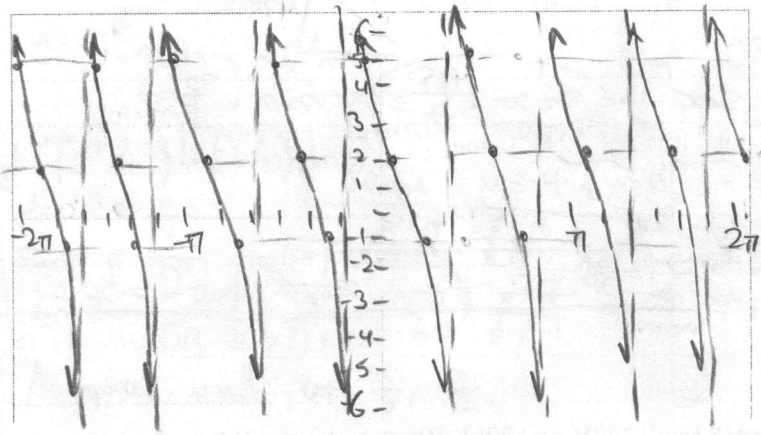
R: $y \in \mathbb{R}$

Find a general formula for all Vert. Asymptote

$x = 4n^R$, $n \in \mathbb{Z}$

7. Graph the function on the graph provided: $y = 3\cot 2\left(\theta - \frac{\pi}{3}\right)+2$

Period: $\frac{\pi}{2}$



Indicate the domain and range:

D: $x \in \mathbb{R}$, $x \neq \frac{\pi}{3} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

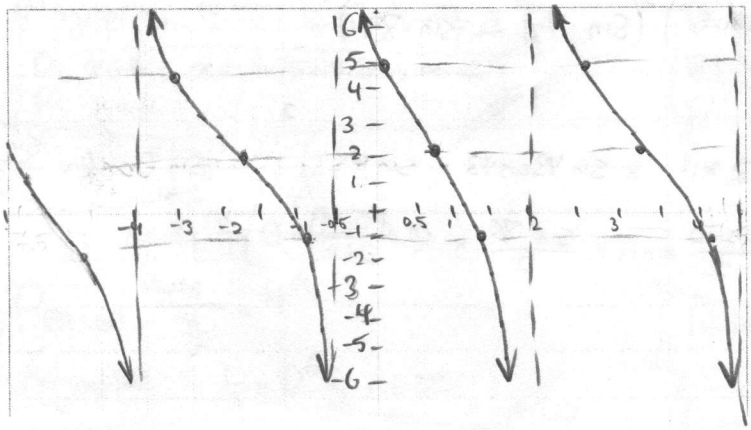
R: $y \in \mathbb{R}$

Find a general formula for all Vert. Asymptote

$x = \frac{\pi}{3} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

8. Graph the function on the graph provided: $y = 3\cot\frac{2\pi}{5}(\theta-2)+2$

Period: $\frac{\pi}{\frac{2\pi}{5}} = \frac{5}{2}^R$



Indicate the domain and range:

D: $x \in \mathbb{R}$, $x \neq 2^R + 2.5n$, $n \in \mathbb{Z}$

R: $y \in \mathbb{R}$

Find a general formula for all Vert. Asymptote

$x = 2^R + 2.5n$

9. Prove the identity: $\dfrac{\tan x}{1-\cot x}+\dfrac{\cot x}{1-\tan x}=\sec x\csc x+1$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}=\frac{\sin^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}=\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}=\sec\theta\csc\theta+1$$ □

10. Suppose $\sec x+\tan x=\dfrac{22}{7}$ Find the value of $\csc x+\cot x=?$ (aime)

$$\frac{1}{\cos x}+\frac{\sin x}{\cos x}=\frac{22}{7}$$

$$\frac{1+\sin x}{\cos x}=\frac{22}{7}$$

$$\frac{1+2\sin x+\sin^2 x}{\cos^2 x}=\frac{484}{49}$$

$$49+98\sin x+49\sin^2 x=484(1-\sin^2 x)$$

$$533\sin^2 x+98\sin x-435=0 \Rightarrow \sin x=-1 \text{ (neg.) or } \sin x=\frac{435}{533}$$

$$\frac{1}{\sin x}+\frac{\cos x}{\sin x}=? \Rightarrow \frac{1+\cos x}{\sin x}$$

plug in

$$=\frac{1+\frac{308}{533}}{\frac{435}{533}}=\frac{29}{15}$$

$\cos x=\dfrac{308}{533}$

11. Find the value of $10\cot\left(\cot^{-1}3+\cot^{-1}7+\cot^{-1}13+\cot^{-1}21\right)$ (aime)

$\cot(a+b)=\dfrac{1-\tan a\tan b}{\tan a+\tan b}$ $\quad a=\cot^{-1}3,\ b=\cot^{-1}7,\ c=\cot^{-1}13,\ d=\cot^{-1}21$

$\tan(a)=\tan\left(\cot^{-1}3\right)=\dfrac{1}{3}$ $\quad \tan b=\dfrac{1}{7}$ $\quad \tan c=\dfrac{1}{13}$ $\quad \tan d=\dfrac{1}{21}$

$$=10\cot\frac{1-\tan(a+b)\tan(c+d)}{\tan(a+b)+\tan(c+d)}=10\frac{1-\frac{\tan a+\tan b}{1-\tan a\tan b}\cdot\frac{\tan c+\tan d}{1-\tan c\tan d}}{\frac{\tan(a+b)}{1-\tan a\tan b}+\frac{\tan c+\tan d}{1-\tan c\tan d}}=10\frac{1-\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\cdot\frac{1}{7}}\cdot\frac{\frac{1}{13}+\frac{1}{21}}{1-\frac{1}{13}\cdot\frac{1}{21}}}{\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\cdot\frac{1}{7}}+\frac{\frac{1}{13}+\frac{1}{21}}{1-\frac{1}{13}\cdot\frac{1}{21}}}=15$$

12. Challenge: Evaluate $(\sin 1^\circ)(\sin 3^\circ)(\sin 5^\circ)\ldots(\sin 177^\circ)(\sin 179^\circ)$ [No Calculators: AoPs]

$\sin(180-\theta)=\sin\theta$

$$=(\sin 1^\circ)^2(\sin 3^\circ)^2(\sin 5^\circ)^2(\sin 7^\circ)^2\cdots(\sin 89^\circ)^2$$

$\sin(90-\theta)=\cos\theta$

$$=\left(\sin 1\cos 1\times\sin 3\cos 3\times\cdots\times\sin 43\cos 43\times\sin 45\right)^2$$

$\sin\theta\cos\theta=\dfrac{\sin 2\theta}{2}$

$$=\left(\frac{\sin 2}{2}\times\frac{\sin 6}{2}\times\frac{\sin 10}{2}\times\cdots\times\frac{\sin 86}{2}\times\frac{\sqrt{2}}{2}\right)^2$$

keep going